Tertium Comparationis
Journal für International und Interkulturell
Vergleichende Erziehungswissenschaft
Vol. 16, No. 1, pp. 76–92, 2010

It's easier said than done: The long way from an educational reform to changes in instruction. A Chilean example of mathematical reasoning and constructivist teaching orientation

Daniela Jiménez
German Institute for International Educational Research (DIPF)

María Leonor Varas
Universidad de Chile, Santiago

Abstract

The implementation of a new national curriculum was one of many strategies in the Chilean educational reform begun in 1990. In mathematics, the new development emphasized mathematical reasoning and incorporated a constructivist teaching orientation, among other issues. This article presents findings from a video-based study that examines mathematical reasoning and constructivist teaching orientation in three-lesson units on the Pythagorean Theorem. The sample comprises 21 7^{th} grade classes, their teachers and correspondent 784 pupils.

The results show that the teachers report a high agreement with constructivist teaching orientation when asked in a questionnaire, but according to the videotaped lessons their teaching rather corresponds to a receptive orientation. Regarding mathematical reasoning, the majority of the teachers introduced the Pythagorean Theorem by means of an inquiry activity, failing to foster mathematical reasoning.

1. Introduction

The implementation of a new national curriculum was one of the several strategies included in the Chilean educational reform begun in 1990. This actualization was a response to the needs of an information and knowledge-based society and aligned to the international trends "according to three criteria: i) changing from an emphasis on contents to skills or competencies; ii) updating and enriching subjects, or requiring higher standards of achievement in them; and iii) ensuring meaning or relevance of the curriculum by pursuing connections to students' lives" (Cox, 2006, p. 44).

Ever since, the Ministry of Education has been carrying out different concrete actions in order to succeed in the implementation of this curriculum in the classrooms, like offering teacher trainings on new contents and didactic orientations, and distributing new textbooks for teachers and pupils.

Since the results of national and international standardized tests have offered broad information about the pupils' learning outcomes, that is, to which extent the curriculum has been achieved by the Chilean pupils, the research agenda has been mainly focused on the learning outcomes and there is only little information about what has actually happened in the classrooms. The most important information source regarding teaching practices in the classroom used to be the evaluation of programs carried out in the context of the reform commissioned by the Ministry of Education (Cox, 2003), whereas the empirical research on instructional issues in specific subject matters is relatively recent.

The aim of the present paper is to contribute to describe teaching practices in Chilean classrooms regarding two aspects fostered by the new curriculum, that is, (1) the promotion of mathematical reasoning and (2) the constructivist teaching orientation (see section 4). Both issues were investigated analyzing 7^{th} grade mathematics videotaped lessons about the Pythagorean Theorem. Because the constructivist teaching orientation is related to beliefs about teaching and learning, teachers' agreement with that perspective is investigated as well. More specifically, the following research questions are examined: (1) how do teachers promote mathematical reasoning when introducing the Pythagorean Theorem, (2) do teachers endorse the constructivist teacher orientation and (3) to what extent do teaching practices reflect a constructivist approach?

Both topics were investigated within the same study; nevertheless, since they represent two main foci, this work is structured by two consecutive thematic sections in order to facilitate reading.

2. Chilean achievement in mathematics: Background

Since 2000–2001, the Programme for International Student Assessment (PISA) evaluates every three years in how far students toward the end of compulsory education have acquired some of the knowledge and skills in language, science and mathematics that are essential for full participation in society. The 2006 PISA test results report that in Reading Literacy, Chile ranks ahead in Latin America with an average score of 442 points (33 more than in the 2000 survey, and 50 points below the Organisation for Economic Co-operation and Development (OECD) average in PISA 2006). Among all test participants, Chile was the country with the greatest score improvements in Reading Literacy since the last survey. In Science, with an average score of 438 points, it ranked 8th among 24 middle-income nations; however, this score was 62 points below the OECD average. In Mathematics, with 411 points, Chile ranked 17th among middle-income nations and 47th amidst the whole group of 57 participant nations (MINEDUC, 2009a).

These poor results in mathematics are even weakened if we consider the score distribution among the six categories that typify the levels of achievement of the PISA test. In the group of fifteen year old Chileans, 55 % scored below 420.1 points. This score corresponds to the upper bound for the lowest level considered by this study (level 1 of 6), typified by direct answer problems, which contain all the necessary information and require elementary actions and operations. More than half of the students (28.2 %) do not even reach the lower bound for this category (357.8 points), so they are below the minimum mathematical knowledge this test intends to assess (ibid.).

Similarly, in the 2003 TIMSS international test, Chilean students' performance in mathematics was comparable to that of countries with a considerably lower Human Development Index. Still further, national tests that systematically assess the learning achievements of 4th, 8th and 10th graders present sustained improvements in language since 2003, but a stall in mathematics (MINEDUC, 2004a).

In short, there is a collection of evidence that shows greater problems in mathematics than in other curricular topics, and even that these have not yet been solved by the many efforts implemented so far.

3. First example: Mathematical reasoning

3.1 Mathematical reasoning at school: Background

Mathematical reasoning has been included as an important topic within mathematics education for several years. In 1989 the National Council of Teachers of Mathematics (NCTM) standards called for a de-emphasis of 'two-column proofs'[1], but still named 'Reasoning' as one of the five core standards that were to be emphasized. At the same time, researchers in mathematics education focused their attention on the role of proving in explaining why a statement is true (Hanna, 1989). In 2000, to clarify and promote the role of justification and proving, 'Reasoning and Proof' was made one of the ten central standards in NCTM's 'Principles and Standards for School Mathematics'.

Likewise, the TIMSS 1999 Video Study[2] included new codes in order to register a variety of special reasoning forms that might be present in eighth-grade mathematics lessons, which might have been omitted by the 1995 coding scheme. Hence, the TIMSS 1999 Video Study's coding scheme for 'Mathematical Reasoning' included *deductive reasoning*, *developing a rationale*, *generalizations*, and *counter-examples* (Jacobs et al., 2003).

> *Deductive reasoning* refers to the derivation of a conclusion from stated assumptions using a logical chain of inferences. There was no requirement that the derivation be formal (e.g., a formal proof), but there was usually an accompanying explanation. *Developing a rationale* was coded when there was an explanation or motivation, in broad mathematical terms, of a mathematical assertion or procedure. … For example, teachers might show that the rules for adding and subtracting integers are logical extensions of those for adding and subtracting whole numbers, and that these more general rules work for all numbers. … *Generalizations* were marked when several examples led to the formulation of an assertion about their shared properties. This process is similar to what many people call inductive reasoning. … Segments were coded as containing a c*ounter-example* [emphases added] whenever an example was used to show that an assertion cannot be true (ibid., p. 119).

Additionally, the first two of eight cognitive mathematical competencies recognized by the PISA mathematical framework refer to mathematical reasoning:

> *Thinking and reasoning*: this involves posing questions characteristic of mathematics ("Is there …?", "If so, how many?", "How do I find ...?"); knowing the kinds of answers mathematics can offer to such questions; distinguishing between different kinds of statements (definitions, theorems, conjectures, hypotheses, examples, conditioned assertions); and understanding and handling the extent and limits of given mathematical concepts.
> *Argumentation*: this involves knowing what mathematical proofs are and how they differ from other kinds of mathematical reasoning; following and assessing chains of mathematical arguments of different types; possessing a feel for heuristics ("What can or cannot happen, and why?"); and creating and expressing mathematical arguments (OECD, 2009, p. 106).

Moreover, the International Commission on Mathematical Instruction (ICMI) Study 19 conference Proof and Proving in Mathematics Education, held in May 2009 in Taipei, recognizes a renewed curricular emphasis on proof worldwide, which has provoked an upsurge in research papers on the teaching and learning of proof at all grade levels. The more than ninety selected papers and invited contributions encompass a variety of views and different aspects showing that the ICMI Study on this topic is both useful and timely. The study defines 'developmental proving' considering three major features (Lin, Hsieh, Hanna & de Villiers, 2009):

- The potential to provide a long-term link with the discipline of proof shared by mathematicians.
- Provide a way of thinking that deepens mathematical understanding and the broader nature of human reasoning.
- Proof and proving are at once foundational and complex, and should be gradually develop starting in the early grades.

To sum up, it is possible to argue that mathematical reasoning and its different expressions are an important topic within mathematics as a school subject matter that is included in current international studies and frameworks for large scale assessment.

3.2 Mathematical reasoning in the Chilean educational system

The national curricular reform for elementary schools in Chile begun in 1996[3] and introduced a new structure in mathematics education, organizing the curriculum in four content strands, namely, *Numbers*, *Arithmetic Operations*, *Shapes and Space*, and *Problem Solving*. At the time, *Problem Solving* was considered a transversal core element, that is, an aspect to be incorporated when teaching any mathematic content (MINEDUC, 2004b).

Lately, the national curriculum has been updated, as a consequence of the regular participation of Chile in international standardized tests like PISA, TIMSS, LLECE and SERCE (UNESCO) and the evolution of the trends in the field of mathematics education, among other reasons. This last curriculum update[4] restructured the four content strands as follows: *Numbers, Algebra, Geometry, Data and Random*, considering *Mathematical reasoning* as transversal core to those strands instead of problem solving, which is now considered a particular instance. In that context mathematical reasoning includes elements such as problem solving, search for regularities and patterns, formulation of arguments and conjectures, modelling of situations or phenomena, among others. Mathematical reasoning is thus rather understood as a means of how to learn mathematics than as content in its own right (MINEDUC, 2009b).

Corresponding to its relevance in the Chilean curriculum, it is not surprising that mathematical reasoning is expected to be a central issue in mathematics textbooks. Therefore, in the textbook evaluation procedure carried out by the Chilean Ministry of Education in order to select those books that will be used in the major part of Chilean schools, issues like understanding of concepts, procedures and logical-mathematical relationships, and presence of proofs and/or justification are included among the main evaluation criteria.

3.3 Teaching mathematical reasoning: Practical issues

As we can see, mathematical reasoning and proving are at the core of doing mathematics, consequently, mathematicians feel comfortable with all the references to mathematical reasoning mentioned previously and they all make sense to them. Nevertheless, since the explicit emphasis on mathematical reasoning and proofs at school level is relatively new, definitions, exact meaning and practical issues relating to mathematical reasoning are insufficiently clear to people who have not experienced doing mathematics[5], which is likely to be the case for most teachers in Chile. Moreover, according to the TIMSS 1999 International Mathematics Report, the Index of Teachers' *Confidence in Preparation to Teach Mathematics* (CPTM) computed with teacher questionnaire data, shows that 45 % of the Chilean students are taught by teachers declaring a low CPTM, while the average of all participating countries reaches 14 % (Mullis et al., 2000).

Possible explanations for this low level of confidence could be their actual preparation to teach mathematics, but also lack of precision and inconsistencies in official documents about how to teach mathematics in accordance with the curricular reform (Cox, 2003).

This might be one of the many reasons why proofs are rarely incorporated in mathematics lessons in Chilean schools. On the contrary, the inquiry methodology for introducing new ideas is the most popular, as it is supposed to allow students to 'discover' important results by themselves. Such is the case with the Pythagorean Theorem that is taught in the 7th grade. As this grade is part of elementary school, the teachers in charge of teaching mathematics at this level are frequently all-purpose teachers instead of specialists in mathematics.

According to the codification used by the TIMSS 1999 Video Study, 'inquiring' forms part of 'mathematical reasoning' and its use should promote a deeper reflection and understanding (Hiebert et al., 2003). This incorporation of inductive thinking – being beneficial – can also confuse students and even the teachers, regarding the value of the deductive method and its essential role in mathematics. With this misunderstanding, it is very frequent to 'discover' theorems without any warning about the limitations of an unproven conjecture. Such lack of precision can have important consequences at the level of generality that this theorem is supposed to hold.

Furthermore, without being prepared on the scientific method, it is rare for the teacher to correctly apply the rules of 'experimental design' within a trial activity. In such a context, conclusions pertaining to this kind of activities may be even more doubtful. For example, if the goal is to discover the Pythagorean Theorem through paper cuttings by comparing the areas of the square built on the sides of the triangle, it is convenient to try with various types of triangles (obtuse, acute, and right triangles) in order to relate the property with the presence of the right angle.

An interesting combination of inductive and deductive reasoning is presented by Jahnke (2009) analyzing the role of mathematical proof in the empirical sciences, recommending that this role should explicitly be discussed in the classroom by means of some exemplary cases. He shows that in the empirical sciences that make use of mathematics we formulate hypotheses about certain phenomena, draw consequences from these hypotheses via mathematical proof and investigate whether the latter fit with the data. In case they do, this speaks in favour of the hypotheses, and we may accept them, or otherwise we reject them. This approach may provide an authentic idea of proof and build a bridge between argumentation in every day situations and mathematical proofs. The author suggests making more use of the term 'hypothesis' that has a broader meaning and is less technical than the term 'axiom':

> The path from every day argumentation to mathematical proof requires a growing consciousness that statements are dependent of other statements and that we cannot speak about truth without specifying the conditions/hypotheses from which we start. Therefore, in school prov-

> ing should be initiated by *inventing hypotheses and experimenting with them* rather then devising chains of deductions (Jahnke, 2009, p. 242).

We point out, as an additional argument in favour of these recommendations, that this is exactly the way mathematicians work and this fits exactly with the new curricular emphasis in mathematics education, namely the requirement that students at all school levels *do mathematics*.

Finally, we would like to stress the relevance of the school topic chosen for this study, namely the Pythagorean Theorem. The Swiss-German research team that originally designed the study (see section 3.4) selected this theorem as an important and challenging school topic which is related to many other mathematical themes (Lipowsky et al., 2008). We point out three additional relevant characteristics of the Pythagorean Theorem related to mathematical reasoning:

- There is no way the student can discover the Pythagorean Theorem without being clearly guided by the teacher.
- The Pythagorean Theorem is just an implication not an equivalence, even though the reciprocal is also true.
- No matter how many inquiry activities are performed or how many explanations are given, the Pythagorean Theorem remains a magical result. The only way of understanding why it is true is by proof.

In this context it is interesting to examine the teaching practices in order to improve mathematical reasoning implemented by Chilean teachers, in particular when introducing the Pythagorean Theorem in the 7^{th} grade.

3.4 Method

The data source for this paper is the Chilean implementation of the core-design of the study ‘Quality of instruction, learning and mathematical understanding’ originally designed and carried out in Switzerland and Germany between 2000 and 2006 (e.g. Klieme & Reusser, 2003).

The Chilean sample consists of 21 mathematics teachers of 7^{th} grade classes and their respective 784 students, who participated in the investigation throughout one school year. Due to its small size, the sample was not intended to be representative but a convenience sample. However, it deliberately included teachers working in different kinds of schools, so the sample included private and public schools, schools with and without state subsidy, different socioeconomic status groups and diverse levels of achievement.[6]

In every class three consecutive lessons about the introduction of the Pythagorean Theorem were videotaped. The content was standardized in order to allow a better comparison and deeper analysis of the lessons.

The teachers did not receive any special information about how to teach, but they were requested to include one proof of the theorem at any moment within the three lessons. Tests of mathematical understanding were submitted to the students immediately before and after the videotaped lesson unit. The participating teachers completed a questionnaire including topics like beliefs about mathematics and teaching mathematics, teaching practices and school features, among others. Besides, teachers answered questions about mathematics content knowledge related to mathematical reasoning. The promotion of mathematical reasoning was examined by analysing the videotaped lessons with an abbreviated version of the video rating system for mathematics didactic aspects developed by Drollinger-Vetter and Lipowsky (2006). This coding scheme included dimensions like structure, abstraction, previous knowledge and proof, among others. Based on the rating scale about proof, an additional scale was developed in order to assess the quality of the inquiry activities. The codification of the videos was made by three trained experts, with high inter-rater reliability.

3.5 Results and discussion

In the first place, Chilean teachers failed to incorporate a proof of the Pythagorean Theorem in their lessons, so it was necessary to apply a test to measure their knowledge in this regard. It is interesting to note that the majority of the teachers believed they had presented a proof through inquiry activity. Furthermore, instructional practices observed in the video footage of classes were not conducive to mathematical reasoning in any of its expressions. The most popular activities of inquiry, designed to 'discover' the Pythagorean Theorem, fail to make their contribution to the development of reasoning. This is due to the avoidance of all aspects of distinction between conjecture and mathematical truth, thesis and assumptions, anecdote and generality. In all of the cases, the inquiry activities considered only right triangles. Consequently, there was no opportunity to test the hypothesis. Moreover, many students had problems checking the result because of sloppy work with the paper cutting task: this had offered an excellent opportunity to prove via a counter-example that the relation between the squares on the sides is not true in general for any triangle, but it was bypassed by the teachers.

An imprecise use of mathematical language did not allow to observe in the videotaped lessons the teacher's knowledge about the logical relations involved in the Pythagorean Theorem, therefore the test for teachers included questions about the theorem statement, its reciprocal and its counter reciprocal statement. The obtained results showed that the teachers were confused about those aspects.

Four of the twenty teachers in the sample were high school mathematics teachers. Contrary to our expectations, these teachers were not more prone to include proofs in their classes than elementary school teachers. A survey about teachers' beliefs on proving (Kotelawala, 2009) shows a similar behaviour in a sample of 78 secondary mathematics teachers in the United States of America. When teachers were compared based on the amount of college mathematics coursework (CMC) they had taken, results indicated that those with more CMC were less inclined to focus on proving. It seems that the mathematics courses at college and university level that are offered to prospective teachers do not include 'proofs that explain' (Hanna, 1989), that is, proofs that allow the student to understand why a statement is true. On the contrary, the prevailing ideas about proof and proving among these mathematics teachers are related with difficult axiomatic exercises and formal rituals that prove without explaining.

Previous analyses with these data carried out by Lacourly and Varas (2009) using hierarchical-linear models showed the impact that the characteristics of teachers and teaching activities could produce in the learning outcomes, controlling previous knowledge and other student and school variables. Specifically, the teacher's knowledge about proofs of the Pythagorean Theorem, the role of proofs in mathematics and the quality of the inquiry instructional activity used to present or 'make-to-discover' the Pythagorean Theorem had a significant impact on students' learning outcomes.

Moreover, a central assumption in the original study is that the quality of the taught proof increases a pupil's understanding of the Pythagorean Theorem, as well as the ability to apply it. The Swiss-German project demonstrated that the quality of the proof that the teacher conducts in lessons is a powerful predictor of learning achievements (Drollinger-Vetter, 2009). We can thus reasonably conclude that it is necessary to improve the ways of teaching mathematical reasoning and proving in school settings. The promotion of mathematical reasoning has not only a theoretical importance, in view of the fact that it contributes effectively to fostering students' mathematical understanding.

4. Second example: Constructivist teaching orientation

4.1 Constructivist and receptive teaching orientation: Background

The constructivist perspective proposes a pupils centred instruction, where knowledge is built upon subjective and social context. That means pupils play an active role in their own learning processes, frequently based on self-regulation, while a teacher's role should essentially be restricted to offering guidance according to pupils' needs, i.e., an adaptive role (Leuchter, Pauli, Reusser & Lipowsky, 2006). From this point of view the interaction teacher student in a constructivist learning environment implies that the teacher offers cognitively stimulating and challenging opportunities to learn that promote the deep understanding of a portion of subject matter (De Corte, 2004). In a constructivist oriented lesson, the pupils' previous experiences (prior knowledge and everyday life) are the basis for a meaningful learning process and mistakes are perceived as an opportunity to learn. In addition, feedback on pupils' performance emphasizes ways of overcoming difficulties and understanding why something is correct or wrong. In mathematics this perspective has been related to problem solving and the promotion of mathematical reasoning (Leuchter et al., 2006).

On the contrary, the direct transmission or receptive teaching orientation is based on behaviourism and, consequently, fosters a teacher centred learning process, that is, learning is understood as the knowledge transmission from teacher to pupils. According to this latter approach, pupils play a passive role, while teachers organize highly structured learning environments using feedback that intends to promote the correct answer, and avoid wrong ones. In mathematics this kind of perspective can include, for instance, learning detailed calculation methods, emphasizing repetition and sometimes memorizing (ibid.).

Constructivist and receptive teaching orientation in Chilean schools

The change in the curriculum for elementary school begun in 1996[7] and incorporated the emphasis on a constructivist teaching orientation in several manners, for instance, in professional development workshops and the didactic orientations included in the mathematics teaching programs. They considered the incorporation of every day life situations in the learning process and emphasis of understanding or problem solving, among others (MINEDUC, 2004b).

In his work on the Chilean new curriculum, Cox (2003) mentions experimenting and learning to learn as essential features, including autonomous learning, questioning, inquiring and an important practical dimension, contrasting hypothesis with real evidence and giving the students opportunities to apply their knowledge in real settings.

Hence, the teacher practices in Chile were expected to change from a more traditional receptive teaching style to a more constructivist one.

According with the evidence from evaluation of programs carried out in the context of the Chilean educational reform, there have been changes in the classrooms settings, especially regarding the relationship between teachers and pupils that is now closer than it used to be. Besides, contemporary learning settings tend to include more aspects from everyday life and the role of the pupils is more active – group work or individual investigation have become usual activities – while the ‘strong teacher’ role is slowly converted to that of a facilitator (ibid.).

It is interesting to note that Bellei (2003) refers to the teaching practices in Chile as being in a transition phase, meaning that they have already begun to shift according to the reform principles but further development is still needed. For example, the pupils now participate in the lessons far more than in the past, but the quality of the participation opportunities must be improved. Many innovative didactic resources are now used in the lessons but they play rather a motivational role and are not clearly oriented to promoting learning (Cox, 2003).

Considering these antecedents it seemed important to investigate the constructivist teaching orientation in two complementary ways: firstly, the endorsement that teachers show regarding the constructivist and receptive teaching orientation was assessed. Secondly, we observed the actual teaching practices in mathematics lessons, in order to find out whether they reflect a constructivist perspective or rather a receptive one.

4.2 Method

The sample was composed of 21 Chilean teachers who participated in the study throughout one school year (see section 3.4 for the full design description).

In order to examine the constructivist teaching orientation, the teachers had to fill in a questionnaire indicating their agreement with different aspects about constructivist and receptive teaching orientation in mathematics. They responded to questions on a 4-point Likert-type scale from 1 (strongly disagree) to 4 (strongly agree). The items were adapted from the teacher questionnaire developed by Rakoczy, Buff and Lipowsky (Klieme, Pauli & Reusser, 2005).

Additionally, videotaped lessons were coded independently by two raters using a 4-point Likert-type scale from 1 (low occurrence) to 4 (high occurrence) using an abbreviated version of the video rating system for quality of instruction developed by Rakoczy and Pauli (2006).

The adapted coding scheme for constructivist teaching orientation was structured into four dimensions and included elements such as the following:

- The exploration of prior knowledge while introducing new contents.
- The pupils are requested to explain a topic according to their own understanding and to answer questions using their own words.
- The teacher poses questions in order to explore the thinking process underlying the pupils' answers.
- The pupils are asked about what they have understood or not.
- The teacher promotes challenging activities in which pupils have to analyze, compare, generalize, or develop hypotheses, among others.
- The interaction between the teacher and the students supports conceptual change and conceptual expansion.

Additionally, the receptive teaching orientation was rated using the same rating format, 4-point Likert-type scale from 1(low occurrence) to 4 (high occurrence), considering aspects such as:

- The teacher gives precise indications about how a problem is to be solved, for instance, he or she recommends a kind of 'recipe' to be applied in all tasks or fosters the use of certain procedures.
- The teacher solves a problem for the whole class that functions as a pattern for the resolution of other ones.

- The teacher usually asks questions that are to be answered with one word or that have exactly one right answer.

4.3 Results and discussion

Although the core topic here is constructivist teaching perspective and not receptive teaching orientation it is interesting to show the results of both scales together and to interpret the information obtained in its entirety. Regarding the agreement with the constructivist teaching orientation, the questionnaire data yielded a mean score of 3.70 (SD = 0.50) showing results clearly above the theoretical average of the response scale (2.50).

In contrast, the scale measuring agreement with the receptive teaching orientation showed a mean of 2.22 (SD = 0.53), slightly below the theoretical average of the response scale (2.50).

The correlation between both scales is r = -.45 (p = 0.04). These results suggest that teachers showing a high agreement to the constructivist teaching orientation tend to show a low agreement with the receptive teaching perspective and vice versa. Such findings seem to be consistent with the theory, since constructivist and receptive teaching orientation are considered quite opposite perspectives. However, it is interesting to note that in a similar analysis carried out in the context of the OECD study 'Teaching and Learning International Survey' (TALIS), only three of the 23 participating countries showed negative correlations between these two perspectives on teaching. The only Latin-American participating countries were Mexico and Brazil, obtaining correlations of 0.74 and 0.65 respectively (Klieme & Vieluf, 2009).

In this context our results suggest that the Chilean teachers of our sample show a relatively clear position in favour of constructivism, meaning if teachers' performance consistent, it should not show a combination of both teaching orientations.

The next step deals with the examination of the teacher performance in the videotaping lessons concerning these perspectives. On the one hand, the results regarding the constructivist teaching orientation show that the occurrence of elements that correspond to this teaching perspective is extremely low. The mean obtained in the dimension *Exploration of prior knowledge* was 1.17 (SD = 0.20), while *Exploration of ways of thought* had a mean score of 1.17 (SD = 0.25). *Challenging problems* and *conceptual change* achieved means of 1.67 (SD = 0.39) and 1.22 (0.36), respectively. These results are clearly below the theoretical average of the response scale (2.50). It is also important to stress that many aspects of the rating system could not be observed at all in any of the three videotaped lessons.

On the other hand, the results in the receptive teaching dimension were quite high, with a mean of 3.59 (clearly above the theoretical average of the response scale 2.50), that is, the occurrence of events related to direct transmission instruction prevailed dramatically during the three videotaped lessons.

It is important to highlight that although many teachers implement activities that in their wording could be challenging to the pupils, the initiative fails in its implementation, for example: after beginning an activity the teacher makes several interventions that reduce the complexity of a task; sometimes giving hints to the student that lie outside the logical chain of the task, but can actually help in getting a right answer. In other cases, complex questions are reduced into a set of questions that can be answered with one word, allowing the teacher to verbalize the conclusion.

It appears to be problematic for the teacher to deal with creative answers or such answers that go beyond the logic of the question. Follow up in these cases is often very poor, sometimes pupils do not receive any feedback at all or the teacher gives the floor to another pupil, just keeping the flow of the classroom discourse. Wrong answers are often ignored, too, even without explicitly saying that they are wrong and the pupil having any opportunity to explain further.

In other occasions regarding low structured tasks, teacher guidance was too poor and pupils seem to be confused about the purpose of the task, for example, they cut squares or triangles but did not know what to do with them. In other cases the teacher asked specific questions about the task and the pupils tended to guess the answers instead of thinking about them. Such a situation might suggest to the teacher that the context of the task is missing and the students did not grasp or forgot the purpose of a hands-on activity. Only a few teachers reminded the students of the context or why they were doing something.

Sometimes pupils are requested to explain their answers in detail, and they actually give detailed answers, but the teacher does not use their contribution at any moment during the lesson and the intervention is foregone.

Anyway, regardless of the teachers' agreement to the constructivist teaching orientation, the observed interactions along the three videotaped lessons corresponded quite clearly to patterns of direct transmission instruction.

Previous studies have shown that kind of discrepancy between beliefs and teaching practice (e.g. Lipowsky, Thussbas, Klieme, Reusser & Pauli, 2003). How can such a discrepancy be explained in our sample?

A possible explanation could be the social desirability when filling in the questionnaire; since teachers know the constructivist teaching orientation is part of the curriculum and teaching programs, they think it is the right thing to do to show agreement if they are asked about that topic. This would mean they do not really agree as strongly as they have reported to.

In addition, it is also a plausible explanation that teachers do agree with constructivist instruction in theory, but when they have to actually design a learning setting or once they have to put it in practice, they fail. Support for this hypothesis can be derived from hard context conditions that impede the implementation according to plan, like problems with classroom management, lack of didactic resources, students absenteeism, continuous time pressure, or other structural difficulties (e.g. Hofer, 1996).

It would be interesting to know what the teachers in our sample think about their performance in the videotaped lessons: maybe the lessons seem to accord with the constructivism to them; or maybe they intentionally tend to use a more receptive teaching style when introducing a new content. Only a longer videotaping period would have allowed us to observe the more constructivist teaching practice. This will remain as an open question.

5. Further perspectives

Due to the small sample of this study the results should be interpreted carefully. In any case, it contributes to identifying some difficulties in the implementation of promoting mathematical reasoning and constructivist teaching orientation in mathematics lessons in Chilean classrooms.

As a result of the examination of teaching practices concerning mathematical reasoning, we could see that the teachers in our sample failed to incorporate proofs in their lessons but included inquiry activities instead. These activities were deficient in distinguishing between conjecture from truth, thesis from assumption and anecdote from generality, impeding a genuine promotion of mathematical reasoning.

Besides, the teaching practices observed corresponded rather to receptive teaching orientation than to the constructivist one although teachers declared their endorsement of the constructivist perspective and a low agreement with the receptive paradigm.

Considering these results, it is possible to conclude that ways to effectively promote mathematical reasoning and constructivist teaching practices must be reinforced in professional development and in initial mathematics teacher training as well. Actually, we think both issues could be linked and conjoined in professional development for mathematics teachers, since activities for the promotion of mathematical reasoning can be easily implemented within a constructivist teaching orientation. Besides, both issues can be simply linked with several mathematics contents at all school levels.

Specifically regarding mathematical reasoning, it seems to be a pending task – especially in initial teacher training – to succeed in incorporating proving as a powerful teaching tool, instead of keeping it as a theoretical issue with doubted practical importance.

Acknowledgement

The findings presented here are part of the study 'Instructional Quality in Mathematics Lessons: Pythagorean Theorem and Mathematical Reasoning' (Análisis de la calidad de clases de matemática: Teorema de Pitágoras y Razonamiento Matemático) partially supported by the Chilean Ministry of Education through its 'Fund for Research and Development' (Fondo de Investigación y Desarrollo) under grant, FONIDE, FIE_000209.

Notes

1. The two column-proof is a kind of proof mostly used in school settings: its name refers to the way it is written, i.e. every step listed in the left-hand column is explained in the right-hand column.
2. The TIMSS Video Studies were carried out in order to examine and compare teaching practices of countries participating in the TIMSS test. For further information see www.llri.org.
3. The first important update of the new curriculum took place in 2002.
4. The last update called 'ajuste curricular' (Curricular adjustment) was elaborated in 2009. The correspondent version of the teaching programs was not officially delivered at the time of this publication.
5. 'Doing mathematics' refers to working with the subject matter the way a mathematician does, for instance, asking himself/herself mathematical questions, trying to verify whether a theorem is valid in cases not considered in their original formulation, developing intuitive ideas, testing them using extreme cases and searching for arguments that allow to prove their validity.
6. The variables were operationalized according to the categories defined by the Ministry of Education of Chile. For details about the sample and the design see Varas, Cubillos and Jiménez (2008).
7. An important update of the new curriculum took place in 2002. The last update called 'ajuste curricular' (Curricular adjustment) was elaborated in 2009. The correspondent version of the teaching programs had not officially been delivered at the time of this publication.

Bibliography

Bellei, C. (2003). Ha tenido impacto la reforma educativa Chilena. In C. Cox (Ed.), *Políticas educacionales en el cambio de siglo*. Santiago: Editorial Universitaria,.

Cox, C. (2003). El nuevo currículum del sistema escolar. In R. Hevia (Ed.), *La educación en Chile hoy*. Santiago: Universidad Diego Portales.

Cox, C. (2006). *Policy formation and implementation in secondary education reform: The case of Chile at the turn of the century* (Education Working Paper Series 3). Washington, DC: Human Development Network, World Bank.

De Corte, E. (2004). Mainstreams and perspectives in research on learning (mathematics) from instruction. *Applied Psychology: An International Review*, *53* (2) 279–310.

Drollinger-Vetter, B. (2009). *Verstehenselemente und strukturelle Klarheit: Anleitung von mathematischen Verstehensprozessen im Unterricht – Fachdidaktische Qualitäten der Unterstützung von Strukturaufbauprozessen während einer dreistündigen Einführung zum Satz des Pythagoras.* Unveröffentlichte Dissertation, Universität Zürich.

Drollinger-Vetter, B. & Lipowsky, F. (2006). Fachdidaktische Qualität der Theoriephasen. In I. Hugener, C. Pauli & K. Reusser (Hrsg.), *Videoanalysen. Dokumentation der Erhebungs- und Auswertungsinstrumente zur schweizerisch-deutschen Videostudie ‚Unterrichtsqualität, Lernverhalten und mathematisches Verständnis‘* (Materialien zur Bildungsforschung, Bd. 15) (S. 206–233). Frankfurt a.M.: GFPF.

Hanna, G. (1989). Proofs that prove and proofs that explain. In G. Vergnaud, J. Rogalski & M. Artigue (Eds.), *Proceedings of the thirteenth conference of the International Group for the Psychology of Mathematics Education* (Vol. II) (pp. 45–51). Paris: PME.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K., Hollingsworth, H., Jacobs, J., Chui, A.M.Y, Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Elterbeek, W., Manaster, C., Gonzales, P. & Stigler, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study*. Washington, DC: U.S. Department of Education. National Center for Education Statistics.

Hofer, M. (1996). Lehrer-Schüler-Interaktion. In F.E. Weinert (Hrsg.), *Enzyklopädie der Psychologie: Praxisgebiete. Pädagogische Psychologie: Psychologie des Unterrichts und der Schule* (S. 213–252). Göttingen: Hogrefe.

Jacobs, J., Garnier, K., Gallimore, R., Hollingsworth, J., Givvin, K., Rust, K., Kawanaka, T., Smith, M., Wearne, D., Manaster, A., Etterbeek, W., Hiebert, J., Stigler, J.W. & Gonzales, P. (2003). *Third International Mathematics and Science Study 1999 Video Study Technical Report. Volume 1: Mathematics.* Washington, DC: National Center for Education Statistics, Institute of Education Statistics, US. Department of Education.

Jahnke, H.N. (2009). Proof and the empirical sciences. In F. Lin, F. Hsieh, G. Hanna & M. de Villiers (Eds.), *Proceedings of the ICMI Study 19 conference: Proof and Proving in Mathematics Education* (Vol. I) (pp. 238–243). Taipei: National Taiwan Normal University.

Klieme, E., Pauli, C. & Reusser, K. (Eds.). (2005). *Dokumentation der Erhebungs- und Auswertungsinstrumente zur schweizerischdeutschen Videostudie ‚Unterrichtsqualität, Lernverhalten und mathematisches Verständnis'. Teil 1: Rakoczy, K., Buff, A. & Lipowsky, F. Befragungsinstrumente* (Materialien zur Bildungsforschung, Bd. 13). Frankfurt a.M.: GFPF.

Klieme, E. & Reusser, K. (2003). Unterrichtsqualität und mathematisches Verständnis im internationalen Vergleich. *Unterrichtswissenschaft*, *31* (3), 194–205.

Klieme, E. & Vieluf, S. (2009). Teaching practices, teachers' beliefs and attitudes. In Organisation for Economic Cooperation and Development (Ed.), *Creating effective teaching and learning environments: First results from TALIS*. Paris: OECD.

Kotelawala, U. (2009). A survey of teacher beliefs on proving. In F. Lin, F. Hsieh, G. Hanna & M. de Villiers (Eds.), *Proceedings of the ICMI Study 19 conference: Proof and proving in mathematics education* (Vol. I) (pp. 250–255). Taipei: National Taiwan Normal University.

Lacourly, N. & Varas, M.L. (2009). Teachers mathematical reasoning does matter. In F. Lin, F. Hsieh, G. Hanna & M. de Villiers (Eds.), *Proceedings of the ICMI Study 19 conference: Proof and proving in mathematics education* (Vol. II) (pp. 47–52). Taipei: National Taiwan Normal University.

Leuchter, M., Pauli, C., Reusser, K. & Lipowsky, F. (2006). Unterrichtsbezogene Überzeugungen und handlungsleitende Kognitionen von Lehrpersonen. *Zeitschrift für Erziehungswissenschaft*, *9* (4), 562–579.

Lin, F., Hsieh, F., Hanna, G. & de Villiers, M. (Eds.). (2009). *Proceedings of the ICMI Study 19 conference: Proof and proving in mathematics education*. Taipei: National Taiwan Normal University.

Lipowsky, F., Thussbas, C., Klieme, E., Reusser, K. & Pauli, C. (2003). Professionelles Lehrerwissen, selbstbezogene Kognitionen und wahrgenommene Schulumwelt. Ergebnisse einer kulturvergleichenden Studie deutscher und Schweizer Mathematiklehrkräfte. *Unterrichtswissenschaft*, *31* (3), 206–238.

Lipowsky, F., Rakoczy, K., Pauli, C., Drollinger-Vetter, B., Klieme, E. & Reusser, K. (2008). Quality of geometry instruction and its short-term impact on students' understanding of the Pythagorean Theorem. *Learning and Instruction*, *19*, 527–537.

MINEDUC (Ministerio de Educación de Chile). (2004a). *Chile y el aprendizaje de matemáticas y ciencias según TIMSS. Resultados de los estudiantes chilenos de 8º básico en el Estudio Internacional de Tendencias en Matemáticas y Ciencias 2003.* Santiago, Chile. Retrieved July 02, 2010, from http://www.simce.cl/index.php?id=103&no_cache=1

MINEDUC (Ministerio de Educación de Chile). (2004b). *Educación matemática programa de estudio séptimo año básico / nivel básico 5*. Retrieved July 02, 2010, from http://www.curriculum-mineduc.cl/docs/fichas/7b09_matematica.pdf

MINEDUC (Ministerio de Educación de Chile). (2009a). *What does PISA tell us about the education of high school students in Chile? New analyses and perspectives on the PISA 2006 results.*

Santiago, Chile. Retrieved July 02, 2010, from http://www.simce.cl/fileadmin/Documentos_y_archivos_SIMCE/PISA2006/PISAinglesLIBRO.pdf

MINEDUC (Ministerio de Educación de Chile). (2009b). *Fundamentos del ajuste curricular en el sector de matemática*. Retrieved July 06, 2010, from http://www.curriculum-mineduc.cl/docs/apoyo/articulo_fundamentos_ajuste_matematica_300309.pdf

Mullis, I.V.S, Martin, M.O, Gonzalez, E.J., Gregory, K.D., Garden, R.A, O'Connor, K.M., Chrostowski, S.J. & Smith, T.A. (2000). *Timss 1999. International Mathematics Report*. Chestnut Hill: Boston College.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

OECD (Organization for Economic Cooperation and Development). (2009). *PISA 2009, assessment framework. Key competencies in reading, mathematics and science.* Paris: OECD. Retrieved July 02, 2010, from http://www.oecd.org/dataoecd/11/40/44455820.pdf

Rakoczy, K. & Pauli, C. (2006). Hoch inferentes Rating. Beurteilung der Qualität unterrichtlicher Prozesse. In I. Hugener, C. Pauli & K. Reusser (Eds.), *Videoanalysen. Dokumentation der Erhebungs- und Auswertungsinstrumente zur schweizerisch-deutschen Videostudie ‚Unterrichtsqualität, Lernverhalten und mathematisches Verständnis'* (Materialien zur Bildungsforschung, Bd. 15) (pp. 206–233). Frankfurt a. M.: GFPF.

Varas, M.L. Cubillos, L. & Jiménez, D. (2008). Análisis de la calidad de clases de matemática: Teorema de Pitágoras y razonamiento. En Ministerio de Educación (Ed.), *Selección de investigaciones primer concurso FONIDE: Evidencias para políticas públicas en educación*, (pp. 123–153). Santiago: MINEDUC.