The following research studies originate from an interdisciplinary research project under the topic "Linguistic Fostering of Mathematical Instruction with Special Emphasis on Multilingualism – Effectiveness of a subject-and language-oriented remedial approach towards linguistic and mathematical processes of understanding, by using vs not using the L1 (Turkish) of the students". The project was granted by the Federal Ministry of Education and Research of Germany (BMBF) for the period of 2014 to 2017 and was conducted by the principal investigators Susanne Prediger (mathematical subproject; Dortmund), Angelika Redder and Jochen Rehbein (linguistic subproject; Hamburg). The studies in this book contribute to the linguistic part of the project.

The data stems from the interventions of 4 Turkish-German bilingual teachers giving 6 lessons each in 7th grade Maths classes of about 90 students, aged 14 to 16, with multilingual biographies, none of whom had received mother-tongue education before in a German school. The students were grouped into 11 subgroups of about 3 to 5 persons each. In accordance with the design, the use of Turkish as the instructional language was increasingly employed during the series of 6 lessons which gave the students time to get accustomed to this new approach. One has to keep in mind that the students, who belong to the second and third immigrant generation, up to then had acquired and practised their L1, or a code-switching variety, with their family and friends only. The mathematical topic was fractions. The students' bilingual abilities in German and Turkish were checked by means of an adapted C-test (s. results given in the appendix). All lessons were videotaped producing 87 hours of recorded material in toto, out of which 35 hours were transcribed according to the semi-interpretative transcription system HIAT by assistance of the computer programme EXMARALDA; the transcripts were translated in an utterance-related format.

Analytical method: For purposes of an in-depth interpretative analysis, we took more than 50 extracts of discourse from the 4th lesson (of the series of 6 lessons), including all 4 teachers and some of the subgroups of students. We analysed the transcribed classroom interactions by applying Functional Pragmatic Discourse Analysis.

Our general hypothesis was that one can enhance mathematical reasoning of multilingual students through teaching them in their L1 (here Turk-

ish) instead of only in the regular instructional language L2 (here German). In this respect, what counted as the criterion of the students' progress was not primarily their proficiency in the mathematical terminology in L2 German, but their increased capacity in verbalizing their cognitive and mental achievements on the basis of the L1 input provided by their teachers. In particular, it is the process of understanding in multilingual classroom discourse by which the students activate supplementary cognitive-mental resources and which, thus, enables the transfer of professional knowledge.

Essential outcome: The improvement of the students' mathematical reasoning especially through means of their understanding in L1 is a new focal point in psycholinguistic, sociolinguistic, didactic, and, last but not least, in linguistic research. The essential outcome of the research presented here is that the enhancement of mathematical reasoning becomes possible via the students' activities of understanding the mathematical instructions in their L1. This is a valid indication of how important it can be not to neglect the resources of the students' multilingual repertoire in an otherwise monolingual institutional frame as is the German school.

In their introductory statements ("Sprachliches Handeln im mehrsprachigen Mathematikunterricht"; §1), *Angelika Redder & Jochen Rehbein* pinpoint the sociolinguistic, psycholinguistic, and functional-pragmatic background of the following research studies on multilingual mathematical discourse.

The study of Jochen Rehbein & Meryem Celikkol on "Multilingual teaching styles and understanding" ("Mehrsprachige Unterrichtsstile und Verstehen"; §2) seeks to give some explanations of the basic conundrum of the project, why the use of L1 Turkish enhances the students' mathematical performance although their L1 is their weaker language compared to their L2 German. Methodologically speaking, preferential emphasis is put on the role of linguistic forms and linguistic functions in the multilingual mathematical discourse. The authors found six multilingual teaching styles - 'style' taken in a pragmatic sense as the form of realization of linguistic action. In their multilingual styles, teachers make use of L1 Turkish in such diverse manners that these match the corresponding varieties of the students' understanding processes. The multilingual teaching styles are related to the stages of understanding in which the students draw on their multilingual repertoire (comprising varieties like L1, L2, code switching, contact Turkish, German as a second language, and mixed varieties) and, thus, serve as a means to boost their cognitive-mental reservoir, e.g. inference making. The students'

multilingual understanding produces structures, procedures, and forms of thinking used for problem and/or task solving activities which differ from language to language. The transcripts reflect the verbalization of this "language of thinking" in L1 as opposed to L2.

The authors distinguish the following six multilingual teaching styles classified according fostering vs. non-fostering effects for mathematical reasoning.

Fostering multilingual teaching styles are:

- Multilingual Languaging by (reciprocal) nexus;
- Multilingual Languaging by Socratic reconsidering
- Multilingual Languaging by direct method

Non-fostering multilingual teaching styles are:

- Multilingual Languaging by replication;
- Multilingual Languaging by staging like in foreign language teaching
- Multilingual Languaging by prioritizing L2 vis-à-vis L1.

In the classroom discourse, the variable structural relations between L1 and L2 make up the diversity of the teaching styles. Fostering styles show a drift towards multilinguality, whereas non-fostering styles show a drift towards L2 German and yield no additional linguistic and/or cognitive support for mathematical reasoning. By means of the fostering multilingual teaching styles, the students' L1 Turkish is liberated of any service function for an L2 German (monolingually oriented) understanding process; rather, by means of the fostering styles, within the mathematical classroom, a multilingual discursive space of action opened up for the students in which they have full access to their multilingual repertoires so that they can develop and extend their cognitive creativity to solve the mathematical problems and tasks.

In their study on "Students' multilingual acting in task-solving discourse" ("Mehrsprachiges Schülerhandeln beim Aufgabenlösen"; §3), Jonas Wagner & Angelika Redder reconstruct how students cope with the multilingual constellation initiated by the teachers. Especially in task- or problem-solving exchanges, classroom discourse will benefit from the cognitive resource multilingualism offers. This holds true because in multilingual settings language and thought intertwine not only in an ordinary, but in a complex way. To comprehend, to consider and to reason by using two or more languages quite often means to mentally represent concepts or ideas in two or more different ways and thus to get a broader picture of them.

The authors focus their analysis on three key stages of the task solving process: the mental search for a solution that follows the introduction and clarification task, the cooperative considerations on a potential solution amongst students and the interactive phase of presenting an attempt for a reasonable and final solution to the teacher. By focussing on these stages and phases, the interactive processing of knowledge – mediated through discursive linguistic action with the teachers and among each other, i.e. through the linguistic-mental and interactive actions - is taken into consideration primarily. Therefore, the line of argument relies on categories such as knowledge modes (i.e. everyday knowledge, experimental or methodological knowledge etc.) and knowledge categories (i.e. concrete single experience, pre-conceptual or conceptual knowledge etc.) as well as the differentiation of an elementary propositional base (epb) and its verbalization in order to grasp the propositional contents of utterances in their discursive processing and to put them in relation to the illocutions the utterances realise

As a result, the authors distinguish four practices (on different levels of discourse) the students use in multilingual teaching and learning settings:

- Adapting to constellative variances (Konstellatives Adaptieren)
- Splitting of linguistic resources along illocutionary aspects (Illokutives Sprachensplitting)
- Multilingual puzzling (Mehrsprachiges R\u00e4tselraten)
- Multilingual exploration (Mehrsprachiges Ausprobieren)

Multilingual students have sensitive meters for assessing the linguistic dimension of a constellation at their disposal. Thus they are able to adjust their own contributions in an effective way, i.e. securing mutual understanding, meeting other interactants' expectations etc. In this constellative adaptations play a role. They take place in different ways initiated by different moments of the constellation: They can be based on linguistic action patterns and thus on corresponding linguistic knowledge or linked to visual or textual material or even based on the speakers' anticipation of the hearers' knowledge, i.e. their mental configurations.

The splitting of linguistic resources along illocutionary aspects (illocutive language splitting) derives from a multilingual access to linguistic action knowledge. The outcome is a verbalisation in which different aspects of the illocutionary force are unfolded via different languages. Illocutive language splittings play a role in activities towards mutual understanding as well as in the transfer or activation of action knowledge, e.g. in cases of

demonstrating linguistically. In both cases, multilingual speech actions by means of illocutive language splitting, above all, serves to foster students' understanding (e.g. of methodical solution steps) and thereby can be made fruitful for the processing of knowledge.

Analyses of meso-level discursive practices such as multilingual puzzling show that even linguistic skills and corresponding linguistic knowledge required in school or classroom discourse (in contrast to everyday or pre-school pragmatic qualifications) prove to be shaped multilingually – at least partly.

Linguistic exploration, also known from but not restricted to language acquisition processes, which is even recognizable in non-fostering multilingual teaching styles, uses all multilingual resources in an attempt to promote understanding. Here multilingual thinking is not only used for discourse management (Arbeitssprache), but also as a language of thought (Denksprache).

Angelika Redder & Jonas Wagner ("Multilingual knowledge processing in free group work", "Mehrsprachige Wissensprozessierung in freier Gruppenarbeit"; §4) address the effectiveness of multilingual action by examining "free language assessments" – mathematical tasks that are thematically independent from the rest of the project's intervention study in slightly different constellations, i.e. with limited influence of the teacher on the development of a solution.

Since the students are required to seek solutions together in independent, cooperative and collective group work, to explain approaches and solutions to each other, to formulate them verbally first and then to consolidate and evaluate them in writing, an insight into the comparatively free handling of their language choices can be reconstructed. Thus, in addition to the contributions in §2 and §3, these discourses present another facet of multilingual learning in German schools.

Generally, the analyses show that the students use their Turkish repertoire as long as they base their deliberations on concrete or materialised objects but use their German repertoire once they generalise (numerically or algebraically) these findings. Supportive actions, such as illustrations, explanations, giving reasons etc., are realised in a similar fashion. They are given in Turkish as long as everyday formats of knowledge (e.g. ordinary empirical knowledge) – in contrast to institutional formats of knowledge such as knowledge gained through teaching and learning in school – are used.

Altogether this mix of languages related to different formats of knowledge draws on the students' overall linguistic skills.

In the final chapter ("Mehrsprachiger Mathematikunterricht: Ergebnis der linguistischen Projektstudien und Perspektiven; §5), *Angelika Redder* gives a summarizing overview over the findings of the book and formulates some requirements and desiderata of further research.